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# A New Gray Hough Transform for Region Extraction from IRS Images

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## Abstract

A technique using Hough transform is described for detection of homogeneous line segments directly from (i.e., without binarization of) gray level images. A definition of "region" in terms of these line segments, with constraints on its length and variance, is provided. The algorithm is able to extract gray regions irrespective of their shape and size. The effectiveness of the method is demonstrated on Indian Remote-sensing Satellite(IRS) images.

Key words : Hough Transform, Satellite image, Region extraction.

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# 1 Introduction

Hough transforms (HT) are used for finding straight lines or analytic curves in a binary image. There are several ways in which HT for straight lines can be formulated and implemented. Risse has listed some of these forms and analyzed them along with their complexities in [1]. The most popular one is  $(\rho, \theta)$  form, which is given by Duda and Hart [2]. This parametric form specifies a straight line in terms of the angle  $\theta$  (with the abscissa) of its normal and its algebraic distance  $\rho$  from the origin. The equation of such a line in x-y plane is,

$$x \cos \theta + y \sin \theta = \rho \quad (1)$$

where  $\theta$  is restricted to the interval  $[0, \pi]$ . This form is used because of its simplicity to understand and ease in implementation.

Hough transforms are applied usually on binary images. Hence, one needs to convert, initially, the gray image to a binary one (through thresholding, edge detection, thinning etc.) to apply HT. Note that, in the process of binarization, some information regarding line segments in the image may get lost. Thus, it becomes appropriate and necessary to find a way of making HT applicable directly on gray images.

This article is an attempt in that direction where, we present a technique for extracting homogeneous regions of arbitrary shape and size in a gray level image based on Hough transform. The regions are defined in term of homogeneous line segments. The technique includes some operations which are performed in a window to obtain homogeneous line segments. For every quantized  $(\rho, \theta)$  cell in the Hough space ( $\rho$  represents radius,  $\theta$  represents angle), the variance of the pixel intensities contributing to the  $(\rho, \theta)$  cell is computed. The cells whose variances are less than a pre-specified threshold, are found. Each such cell would represent a homogeneous line segment in the image. The window is then moved over the entire image, so as to result in an output consisting of only the homogeneous line segments; thereby constituting different homogeneous regions. The performance of the method has been demonstrated on Indian Remote-sensing Satellite (IRS) images for different parameter values.

In this connection we mention the methods of gray scale Hough transform (GSHT)

of Lo and Tsai [3], and generalized Hough transform (GHT) of Ballard [4]. GSHT enables one to find thick lines (called bands) from gray scale images. Therefore, it can be used for detecting road like structures only in remote-sensing images. GHT is able to extract arbitrary shapes from the edge map of a gray image using prototype information of the objects to be extracted. Note that our method does not need this information and is thus able to extract objects of irregular shapes and arbitrary sizes as found in remote sensing images.

## 2 Definition and Formulation

A region in a gray image can be viewed as a union of several line segments, so that it consists of a connected set of pixels having low gray level variation. Therefore, to extract a region, we need to define a line segment in gray level image. A line segment in a gray level image is defined using two threshold parameters, minimum length of the line ( $l$ ) and maximum variation of the line ( $v$ ). The mathematical formulation of the region in terms of line segments is stated below.

**Def. 1:** A pixel  $P = (i, j)$  is said to fall on a line segment joining the pixels  $P_1 = (i_1, j_1)$  and  $P_2 = (i_2, j_2)$  if  $\exists \lambda_0, 0 \leq \lambda_0 \leq 1$  such that

$$\lambda_0 P_1 + (1 - \lambda_0) P_2 = P.$$

**Def. 2:** A collection of pixels  $L(l, v)$  is said to be a line segment in a gray image if,

there exist  $P_1 = (i_1, j_1)$  and  $P_2 = (i_2, j_2)$ ,  $P_1 \neq P_2$ ; such that

$L(l, v) = \{ P : P \text{ is a pixel falling on the line segment joining } P_1 \text{ and } P_2 \},$

- The number of pixels in  $L(l, v)$  is  $\geq l$ , and
- The variance of the gray values of pixels in  $L(l, v) \leq v$ .

Let  $\mathcal{A}_{l,v} = \{L(l, v) : L(l, v) \text{ is a line segment in the image}\}$ . That is  $\mathcal{A}_{l,v}$  represents the collection of all line segments in the image.

**Def. 3.1:** A pixel  $P$  is said to be in the homogeneous region if  $P \in L(l, v)$ , at least for one  $L(l, v) \in \mathcal{A}_{l,v}$ .

**Def. 3.2:** A pixel  $P$  is said to be in the non-homogeneous region if  $P \notin L(l, v)$ ,  
 $\forall L(l, v) \in \mathcal{A}_{l, v}$ .

Let  $N_H = \{P : P \text{ is a pixel in the non-homogeneous region}\}$ .

The region  $R$  is defined as follows.

**Def. 4.1:** Let  $L_1(l, v)$  and  $L_2(l, v) \in \mathcal{A}_{l, v}$ . Then  $L_1(l, v)$  and  $L_2(l, v)$  are said to be directly connected if either  $L_1(l, v) \cap L_2(l, v) \neq \phi$  or  $\exists$  pixels  $P_1 \in L_1(l, v)$  and  $P_2 \in L_2(l, v)$  such that  $P_1$  is one of the eight neighbours of  $P_2$ .  
Note that a line segment  $L(l, v)$  is directly connected to itself.

**Def. 4.2:** Two line segments  $L_\alpha(l, v)$  and  $L_\beta(l, v)$  belonging to  $\mathcal{A}_{l, v}$  are said to be connected if they are directly connected or there exist  $L_i(l, v) \in \mathcal{A}_{l, v} : i = 1, 2, \dots, k$  where  $k \geq 3$  such that  $L_i(l, v)$  and  $L_{i+1}(l, v)$  are directly connected  $\forall i = 1, 2, \dots, (k-1)$   
where  $L_1(l, v) = L_\alpha(l, v)$  and  $L_k(l, v) = L_\beta(l, v)$ .

**Def. 4.3:** Let  $B_{L_\alpha}(l, v) = \{L(l, v) : L(l, v) \in \mathcal{A}_{l, v} \text{ and } L(l, v) \text{ and } L_\alpha(l, v) \text{ are connected}\}$ .  $L_\alpha(l, v) \in \mathcal{A}_{l, v}$ .

Note that for  $L_\alpha(l, v), L_\beta(l, v) \in \mathcal{A}_{l, v}$  either

$$B_{L_\alpha}(l, v) = B_{L_\beta}(l, v) \text{ or } B_{L_\alpha}(l, v) \cap B_{L_\beta}(l, v) = \phi$$

Note also that  $\bigcup_{L_\alpha(l, v) \in \mathcal{A}_{l, v}} B_{L_\alpha}(l, v) = \mathcal{A}_{l, v}$ .

That is  $\mathcal{A}_{l, v}$  is partitioned into finitely many sets using  $B_{L_\alpha}(l, v)$ 's.

**Def. 4.4:** Let  $R_{L_\alpha}(l, v) = \bigcup_{L(l, v) \in B_{L_\alpha}(l, v)} L(l, v)$ . Then  $R_{L_\alpha}(l, v)$  is said to be a region generated by  $L_\alpha(l, v)$ . Note that  $R_{L_\alpha}(l, v)$  is a set consisting of pixels in the given image. Observe also that the same region can be generated by different line segments (follows from Def. 4.3)

## 2.1 Observations on the above definitions

(a) A line segment may be termed as a *region* according to the above definitions.

- (b) The variance of  $n$  points  $x_1, x_2, \dots, x_n$ , is given by  $\frac{1}{n} \sum (x_i - \bar{x})^2$  where  $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$ .  
Now, variance  $< v \implies \sum (x_i - \bar{x})^2 < nv$ .

Observe that there may be a point (say,  $x'$ ) among  $x_1, x_2, \dots, x_n$ , such that  $(x' - \bar{x})^2$  may be high (say,  $(x' - \bar{x})^2 > kv$ , where  $k > 1$ ). Even then  $\sum (x_i - \bar{x})^2$  can still be less than  $nv$ . This observation indicates the removal of noise upto some extent by the proposed variance based definition of line segment.

- (c) The values for  $l$  and  $v$  are to be chosen "appropriately" to obtain the actual regions in an image. Some portions of the actual regions may not be obtained if the value of  $l$  is high. If the value of  $v$  is high then some spurious collection of pixels may be termed as a region. If the value  $v$  is low then some actual regions in the image may not be obtained. Similarly if the value of  $l$  is low then some unwanted regions may arise.
- (d) Observe that a pixel  $P$  does not fall into any region  $\implies$  There exists no line segment passing through  $P$  with low gray level variation (i.e. variance of the gray level values of the pixels on any line segment passing through  $P$  is greater than  $v$ ). Hence the above stated definitions intend to suppress the pixels in the non-homogeneous region of the image.
- (e) Note that two adjacent collections of connected pixels, each collection of pixels having some gray level value but the gray level value for one collection is distinctly different from the gray level value of the other, may fall into the same region (thus losing their identity) according to the definitions. A further processing of this region will provide the above stated two collections distinctly. (One of the simplest ways of obtaining the two collection of pixels is to apply the gray level thresholding methodology on the region thus obtained.) ♠

The definitions regarding *regions* have been stated above. There may exist several other ways of obtaining the said regions from the image. Note that regions have been defined as a union of line segments, and Hough transform is a standard method of obtaining line segments. But the Hough transform finds line segments in a binary image. In order to make HT applicable directly on a gray image, we formulate a method in the next section which is able to find line segments (and hence the regions) in a gray image.

## 2.2 Strategies of region extraction in gray level image using Hough transform

*Extraction of line segments:*

- (i) Consider the equation for a straight line to be  $x \cos \theta + y \sin \theta = \rho$ . Apply suitable sampling on  $\theta$  and  $\rho$ , and construct the Hough accumulator. Transform each point of the image (pixel) using different values of  $\theta$  (and its corresponding  $\rho$  values). Note that a point in the image space is mapped to more than one cell in the Hough space and each of these cells represents a line in the image space.
- (ii) Compute, for each cell, the length of the corresponding line ( $L$ ) as the total number of image points (pixels) mapped into that cell, i.e., the cell count. Variance of the said pixel values may be termed as the variance ( $V$ ) of the corresponding line.
- (iii) For a cell in the Hough space if the length of the line is less than  $l$  or variance of the line is greater than  $v$  then suppress the cell in the Hough space.
- (iv) Remap this Hough space (containing unsuppressed cells) to image space. This process of remapping preserves all those pixels which are not suppressed in at least one of the cells of the Hough space. Since this transformation preserves only the location of pixels, not their gray values, they may be restored from the original image.

Let us consider an image of size  $M \times M$ . If  $M$  is too large compared to the threshold parameter  $l$ , then  $L$  values (cell counts) will be larger, and as a result, variance of the gray levels on a line may exceed the threshold value  $v$ . Many genuine line segments, therefore, may not be detected in such a case. To avoid this, the search process for obtaining the line segments is to be conducted locally. That is, a window of size  $\omega \times \omega$  needs to be moved over the entire image to search for line segments. Here  $\omega$  may be taken as,  $2l > \omega \geq l$ , because  $\omega \geq 2l$  may still lead to the suppression of actual lines in the image.



### *Extraction of Regions:*

One can clearly see that the above mentioned process extracts line segments which are connected according to the *Definition 4*. Therefore the collection of these line segments will result in regions of different sizes and shapes.

### *Note:*

- There is no restriction on the shape of the "region" thus obtained. The only restriction, we used on the size of the region (i.e., length of the line  $\geq l$ ), is a weak one.
- The method does not need any prior representation of the shape of region to be detected. Therefore, it can extract regions of arbitrary shape and size.

## 3 Algorithm and Implementation

It has been mentioned in the earlier section that values for  $l$  (length of the line),  $v$  (variance threshold) and  $\omega$  (window size) are to be selected and the obtained lines are to be remapped to image domain to procure regions. This process of remapping the cells in the Hough space is to be carried out on every window. The steps of the entire algorithm are stated below.

Step 1: For a window ( $\omega \times \omega$ ) of the image obtain the Hough accumulator values for different  $\rho$  and  $\theta$ .  $\rho$  values and  $\theta$  values are sampled suitably in their respective domains. For each cell in Hough space, mean and variance of corresponding pixel values in the image domain are computed using two more accumulators (one for sum of gray values  $\sum x$  and one for sum of squares of gray values  $\sum x^2$ ). These sum and sum of squares along with the count of cells ( $L$ ) will be used for computing the variance  $V = \frac{\sum x^2}{L} - \left(\frac{\sum x}{L}\right)^2$  of the cell. If the cell count is  $< l$  then replace the cell count by zero (i.e., the cell is suppressed). If  $V > v$  then also replace the cell count by zero. The cells with count non-zero are remapped to image domain preserving the position of window.

Step 2: Repeat Step 1 for all possible windows of size  $\omega \times \omega$  in the image.

Step 3: Restore the gray values of remapped pixels from the original image.

The number of computations in the aforesaid algorithm can be reduced drastically in the following way.

- (i) Note that for a given window size  $\omega$ , Hough transformation of the pixel does not change with its location, because the reference frame for computing  $\theta$  (and hence  $\rho$ ) remains the same. Thus, the Hough accumulator values can be calculated only once and be used for every position of the window.
- (ii) Again, all possible windows of size  $\omega \times \omega$  need not be considered. The window can be moved by half of its size, both horizontally and vertically. This process, though marginally reduces the accuracy of the regions obtained, decreases computations drastically.
- (iii) Keeping the point (iv) of section 2.2 in mind, the process of restoring gray values in the aforesaid Step 3 can be combined with Step 1 by preserving only those pixels in the image domain which are not getting suppressed in at least one cell of the Hough space.

## 4 Results

We have applied the proposed method on IRS (Indian Remote-sensing Satellite) images to demonstrate its usefulness. The IRS images considered here have spatial resolution of  $36.25\text{m} \times 36.25\text{m}$ , wavelength range  $0.77\mu\text{m} - 0.86\mu\text{m}$  and gray level values in the range 0-127 [5]. Size of the images is  $512 \times 512$ . An enhanced (linearly stretched) image is provided in Fig. 1.a (city of Calcutta) and Fig 2.a (city of Bombay) for the convenience of readers, since the original images are poorly illuminated. However the method has been implemented on the original images.

In the present investigation, we have used  $w = 16$  and  $l = 14$  for various values of  $v$ . The output corresponding to Fig 1.a and 2.a for  $v = 0.2, 0.4$  and  $0.6$  are shown in Figs.

1.b, 1.c and 1.d, and Figs. 2.b, 2.c and 2.d respectively. As  $v$  increases, the number and size of detected regions, as expected, increase. Note that the set of pixels in Fig. 1.b is a subset of that of Fig. 1.c, similarly Fig. 1.c is a subset of Fig. 1.d.

One can see that some of the vegetation and habitation areas along with some roads of Calcutta, which are visible in Fig. 1.d, were not detected for  $v = 0.2$  (Fig. 1.b) and  $0.4$  (Fig. 1.c). This is also true for the Bombay image. The river Ganges in Calcutta image has come out in all the cases as a single region, except for  $v = 0.2$  where the two bridges on the river were detected as different regions. On the other hand, the bridge on the creek water in Bombay image is found to be visible for all the values of  $v$ , although the detected sea for  $v = 0.2$  is not as smooth as in the cases of  $v = 0.4$  and  $0.6$ . Experiment was also conducted for other values of  $l$  such as  $l = 15$  and  $16$ , but the results were not much different.

## 5 Conclusions and Discussion

A method of extracting regions in a gray level image using the principle of Hough Transform has been described. A definition of "region" in terms of line segments is provided. Since the methodology does not involve any representation (such as parametric, template etc.) of the shape of regions, it has the ability to detect regions of arbitrary shape and size.

To restrict the size of the article, we have presented the results corresponding to  $v = 0.2, 0.4$  and  $0.6$  for  $\omega = 16$  and  $l = 14$  only, although the experiment was also conducted for other values of  $l$  (e.g.,  $15$  and  $16$ ) and  $\omega$  (e.g.,  $8, 24$  and  $32$ ). Variance of pixel values is used here as a measure of homogeneity of line segment. One may use any other homogeneity measure for this purpose.

Note that selection of  $v$  is crucial. For an image with greater variance, one needs a high value of  $v$  for meaningful interpretation of the detected regions.

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Fig. 1.a: Input IRS image for Calcutta



Fig. 1.b: Output with  $v = 0.2$



Fig. 1.c: Output with  $v = 0.4$



Fig. 1.d: Output with  $v = 0.6$



Fig. 2.a: Input IRS image for Bombay



Fig. 2.b: Output with  $\nu = 0.2$



Fig. 2.c: Output with  $\nu = 0.4$



Fig. 2.d: Output with  $\nu = 0.6$